Random signed measures constructed using Euler characteristics

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Motivated by the connectome data generated by the Blue Brain Project, we wish to define random signed measures over a sigma algebra containing subsets of Euclidean space when the stochastic input is a random abstract simplicial complex, \$K\$, alongside a random location map for the vertices \$f:V \to \mathbb{R}^d\$ (where \$V\$ is the set of vertices in \$K\$). Loosely speaking, we want the measure of \$A \subset \mathbb{R}^d\$ to be related to the Euler characteristic of an appropriate subset of \$K\$ living in \$A\$. We explore two different paradigms where the sigma algebra for our random measures is finite, and when it is the Borel sigma algebra. The allowable construction methods for creating random signed measures differ in these two different paradigms. We can use Euler characteristics with compact support when we have made finiteness restrictions on our sigma algebra but not in the Borel sigma algebra case. In both paradigms we can use a modification of Euler characteristic which includes a weighting for each cell. We also check that our construction of random signed measures is a measurable function (which allows us to define expectations, variances and co-variances).

This is joint work with Victor Panaretos.